Chaotic, Non-Linear Dynamics of Water Chemistry in a Florida Freshwater Stream

Danielle L. Watts, Ray Huffaker, and Matthew J. Cohen

UNIVERSITY OF FLORIDA ECOHYDROLOGY LABORATORY

Introduction

- Chaotic systems are non-linear, and exhibit strong sensitivity to initial conditions. While inherently deterministic, modeling in chaotic systems leads to increasing divergence from expectations over time.
- Chaotic systems also have a strange attractor (fractal correlation dimension)
- Florida Springs are increasingly being understood as complex systems.
- Here we investigate metrics of river metabolism for chaotic behavior and reconstruct the data using a single spectrum analysis.

Chaos Theory is most commonly understood as the butterfly effect, where small changes in a complex system can lead to large and unexpected results.

Field Sampling

- Sensor arrays were deployed in the Ichetucknee River just above US 27 in 2010.
- NO₃ was measured using an In Situ Ultraviolet Sensor (ISUS) version 3 at 15 min intervals.
- Dissolved oxygen (DO) was measured hourly by a YSI 600XL meter.
- The longest data sets likely to have sufficient oscillations (diel cycle) to provide accurate results were chosen (2 for DO, 13 for NO₃).

Chaos and Fractal Dimension

Examples of the reconstructed strange attractor.

<table>
<thead>
<tr>
<th>Time Series</th>
<th>Surrogate (mean)</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>DO mg/L</td>
<td>Correlation Dimension</td>
<td>2.98</td>
</tr>
<tr>
<td></td>
<td>Lyapunov Exponent</td>
<td>1.77</td>
</tr>
<tr>
<td>DO mg/L</td>
<td>Correlation Dimension</td>
<td>1.90</td>
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<td></td>
<td>Lyapunov Exponent</td>
<td>4.25</td>
</tr>
<tr>
<td>NO₃ mg/L</td>
<td>Correlation Dimension</td>
<td>0.92</td>
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<tr>
<td></td>
<td>Lyapunov Exponent</td>
<td>6.45</td>
</tr>
<tr>
<td>NO₃ mg/L</td>
<td>Correlation Dimension</td>
<td>1.73</td>
</tr>
<tr>
<td></td>
<td>Lyapunov Exponent</td>
<td>3.28</td>
</tr>
</tbody>
</table>

Examples of the chaos measures for some of the data sets.
- All of the DO and SpC (not shown) data sets exhibited a significant Lyapunov Exponent and correlation dimension.
- Comparitively, only about half of the nitrate data sets had both significant chaos and correlation dimension, although the L E. was always positive (ranged 1.93-6.45) with a fractal correlation dimension (ranged 0.93-2.73).

Methods

Chaos

The Lyapunov Exponent (L.E.) is a measure of behavior near the edge value of equilibrium and is one approach to show deterministic chaos. This exponent was calculated and then compared to 50 surrogate data sets generated by both a noisy limit cycle and the modified amplitude adjusted fourier transformation to ensure noise did not generate a false positive L.E. We then used an embedding delay to reconstruct the strange attractor, and similarly compared the fractal dimension to surrogate data sets.

Single Spectrum Analysis

SSA is a technique to extract trends and cycles from data sets using an embedding delay. To ensure the strange attractor was not caused by red noise, a Monte-Carlo SSA technique was employed. Red noise is a decaying autocorrelation structure in delay. To ensure the strange attractor was not caused by red noise, we then used an embedding delay to reconstruct the data using a single spectrum analysis.

Conclusions

- Results indicate:
  - Strongly oscillatory components with some trend, where red noise is not a significant component.
  - A well defined attractor for all three metabolic constituents (NO₃, DO, and SpC), and
  - Fractal and chaotic nature to the attractors.

- Chaos in metabolism may be the result of weather influence—induced by chaos in cloud cover and precipitation patterns, etc.
- The unexpected 12 hr cycle may be caused by upstream influences on local conditions.
- Spring metabolism is non-stationary, and these results provide the skeleton framework for modeling.