Numerical Simulation of Variable-density Groundwater Flow: Submarine Groundwater Discharge

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Submarine Groundwater Discharge (SGD)

Schematic depiction of processes associated with SGD. Arrows indicate fluid movement.
- It is an unseen persistent phenomenon;

- low specific rates which make detection and quantification difficult;

- react with sediment components, thus discharging fluids are chemically and ecologically important to the coastal ocean, due to the potential impacts resulting from contaminants or nutrients carried in groundwater.

- It’s a density-dependent flow.
# Diving Forces

<table>
<thead>
<tr>
<th>Components</th>
<th>Driving Forces</th>
<th>Contributing Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meteoric Water (fresh)</td>
<td>Hydraulic gradient</td>
<td>Topography, Transmissivity, Precipitation, Evapotranspiration</td>
</tr>
<tr>
<td>Recirculated seawater (salt)</td>
<td>Hydraulic gradient, tidal pumping, Sea Wave</td>
<td>Tidal range, Period, frequency, Wind force, direction</td>
</tr>
<tr>
<td>Connate waters (very salty)</td>
<td>Density, Thermal gradient</td>
<td>Geology, Geothermal heating</td>
</tr>
</tbody>
</table>

(Burnett, et al. 2003)
Measuring Methods (Burnett, et al. 2006)

- Seepage meters
- Piezometers
- Natural tracers
- Water balance approaches
- Hydrograph separation techniques
- Theoretical analysis and numerical simulations
Variable-density Groundwater Flow

- Field observations and mathematical analyses have shown that the relatively minor variation in groundwater density has a substantial effect on groundwater flow rates and patterns.

- The water velocity as a result of the flow problem is a parameter in the solute transport problem (standard case) and the solution density as a parameter in the flow problem is dependent on concentration, result of the transport problem (specific for variable-density flow).
SGD is a variable-density flow problem which needs to be solved by coupling water flow and solute transport equations:

→ highly non-linear
→ difficult to find a suitable initial guess
→ the behavior of the solutions as functions of the parameters can be very complicated
Because the basic flow is modified by density variations, the numerical model is the only way to understand and predict mass transport at high concentrations.

The analysis and interpretation of observations are much more difficult than in groundwater flow and transport at low concentrations.

The complication caused by geological properties of aquifer, ambient environment and topography, climatic factors, tides and waves, or artificial structures enhance greatly the difficulty of performing an accurate modeling in coastal areas.
It is necessary to consider that SGD includes two components: the net fresh groundwater discharge, which is the flux of terrestrial groundwater toward the sea and the recirculated seawater discharge. The mechanisms includes tidal pumping, near shore circulation due to tides and waves, seasonal exchange or saline circulation.
Methodology

■ The variable-density groundwater flow equation is developed in terms of equivalent freshwater head and fluid density for better uniting MODFLOW and MT3DMS;

■ The movement of groundwater and the transport of solutes in the aquifer are coupled processes;

■ Fluid density is predominantly affected by the solute concentration and fluid pore pressure.
**Concept of Equivalent Freshwater Head**

\[
\begin{aligned}
\begin{cases}
    h_f &= \frac{P_N}{\rho_f g} + Z_N \\
    h &= \frac{P_N}{\rho g} + Z_N
\end{cases}
\end{aligned}
\Rightarrow
\begin{aligned}
\begin{cases}
    h_f &= \frac{\rho}{\rho_f} h - \frac{\rho - \rho_f}{\rho_f} Z \\
    h &= \frac{\rho_f}{\rho} h_f + \frac{\rho - \rho_f}{\rho} Z
\end{cases}
\end{aligned}
\]

Piezometers represent equivalent freshwater head
Governing Equation for Flow in Terms of Freshwater Head

\[
\frac{\partial}{\partial x} \left( \rho K_{fx} \frac{\mu_f}{\mu} \left[ \frac{\partial h_f}{\partial x} \right] \right) + \frac{\partial}{\partial y} \left( \rho K_{fy} \frac{\mu_f}{\mu} \left[ \frac{\partial h_f}{\partial y} \right] \right) + \frac{\partial}{\partial z} \left( \rho K_{fz} \frac{\mu_f}{\mu} \left[ \frac{\partial h_f}{\partial z} + \frac{\rho - \rho_f}{\rho_f} \right] \right) \\
= \rho S_f \frac{\partial h_f}{\partial t} + \theta E \frac{\partial C}{\partial t} - \rho_s q_s 
\]

\( K_f(x, y, z) [LT^{-1}] \): freshwater hydraulic conductivity,
\( \mu_f / \mu \):is considered equal to 1,
\( S_f [L^{-1}] \):specific storage in terms of freshwater head,
\( \theta \):porosity,
\( \rho_s [ML^{-3}] \) & \( q_s [T^{-1}] \):the sources and sinks.
Governing Equation for Solute Transport

Concentration and Density (isothermal conditions)

\[
\frac{\partial C}{\partial t} = \nabla \cdot (D \cdot \nabla C) - \nabla \cdot (\vec{v}C) - \frac{q_s}{\theta} C_s \quad (3)
\]

\(D \ [L^2T^{-1}]:\) hydrodynamic dispersion coefficient,

\(\vec{v} \ [LT^{-1}]:\) fluid velocity,

\(C_s \ [ML^{-3}]:\) solute concentration from sources or sinks

\[
\rho = \rho_f + EC \quad (4)
\]

\[
\frac{\partial \rho}{\partial C} = E \quad \text{(i.e. } E = \frac{\rho_s - \rho_f}{C_s - C_f} = \frac{1025 - 1000}{35 - 0} = 0.7143)\)
**Model Set-Up**

- **Constant head**
  - North
  - No flow
  - South

- **Time-dependent head**

- **24 hours Tidal Cycle**

- **Stages**:
  - (a) Low Tide
  - (b) Flood Tide
  - (c) High Tide
  - (d) Ebb Tide
Numerical Model

- **SEAWAT-2000** (Guo and Langevin, 2002)
  - combining a modified version of MODFLOW2K and MT3DMS;
  - finite difference method;
  - simulation of three-dimensional, variable-density, transient ground-water flow in porous media.
Influence of Tides

Tide is no that regular (ex. quasi-cosinoidal).
Salinity Distribution along the Seashore
Simulation Results

Flow Patterns of SGD – A 4-Step Cycle

1. At low tide, positive terrestrial hydraulic gradient, similar to a classic static-sea-level model.
2. During flood tide, seawater infiltrates into the aquifer (SGR), and SGD gradually decreases as the opposing hydrostatic pressure increases.
3. At high tide, the hydraulic gradient is now reversed, SGD approaches zero, and the SGR mixes with the fresh groundwater in the aquifer and flows inland.
4. During ebb tide, the positive hydraulic gradient recovers, and thus the aquifer releases a large amount of mixed water.
Tidal Amplitude
Compare to Direct Observations

Observed Salinity Profiles

Simulated Concentration Profiles

Jun 27-29, 2006

Oct 19-21, 2006
### Table 3

**Major Uncertain Factors and Sensitivity Investigations in this Study**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Uncertainty in the Study</th>
<th>Sensitivity Investigation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heterogeneity</td>
<td>Anisotropy, stratification, stochastic permeability distribution</td>
<td>–</td>
<td>Simmons et al. 2001</td>
</tr>
<tr>
<td>Aquifer thickness</td>
<td>Assumption error</td>
<td>Doubling the aquifer thickness leads to an significant increase on both fresh and marine SGD</td>
<td>Taniguchi et al. 1999</td>
</tr>
<tr>
<td>Seepage face on the beach</td>
<td>Exists but cannot be modeled by SEAWAT2000</td>
<td>No significant influence in this study, but will affect local water table variation</td>
<td>Li et al. 1997</td>
</tr>
<tr>
<td>Beach slope</td>
<td>Varies no more than 0.3° (geometry mean) per year</td>
<td>No significant influence in this study, but will strongly affect the IMZ when the variation per year is larger than 2°</td>
<td>Mao et al. 2006</td>
</tr>
</tbody>
</table>

1Reference indicates particular study that researched the same variable in different coastal environments. Dash indicates that an investigation was not performed.
Conclusions

- The increase of the tidal amplitude will lead to the increase of total SGD, which is mainly from intensified seawater recirculation.
- This simulation indicates that the percentage of fresh SGD related to the total SGD ranges from 4% to 50% at most under normal conditions.
- Freshwater discharge is only a small part of the total SGD at this site.
- The mixing zone profile for a transient flow under tidal pumping influence can be quite different from the pattern described by the Ghyben-Herzberg relationship especially in the intertidal region.
- The shape and distribution of the intertidal mixing zone is mainly controlled by tidal pumping.